NEW YORK QUILT collector Joanna Rose received her first quilt, a red and white blanket with a schoolhouse design, upon the birth of her first child in 1957. She hasn’t stopped collecting since and has amassed more than 1,300 quilts. When Ms. Rose, who belongs to a prominent New York real-estate family, recently turned 80, she decided to put her entire collection of red and white quilts—some 650—on public display.

Hosted by the American Folk Art Museum, “Infinite Variety: Three Centuries of Red and White Quilts” at the Park Avenue Armory, beginning next Friday and ending March 30, will be the largest textile exhibition ever presented in New York City.

Red and white is a classic color combination in quilt making, popular since the early 19th century as a result of the remarkable colorfastness of Turkey red dye, discovered in the Mediterranean and derived from madder root. Ms. Rose’s quilts, made in a variety of patterns and designs, span the 19th to the 21st centuries. No two are identical, and most bear the unique signatures of their makers, sewn neatly at the corners. Elizabeth V. Warren, the exhibit’s co-curator, discovered that two of the quilts, bought separately years apart, were created by the same quilter.

To display the collection, the exhibit’s curators decided on a strikingly modern design. The American Folk Art Museum held a competition for the quilts’ display, inviting submissions from a range of design firms, and ultimately decided on Thinc Design, a New York-based firm that is known for contemporary work. The company is responsible for the interiors of the National September 11 Memorial & Museum and the Beijing Olympic Games Pavilion.

Thinc’s design organizes the quilts into tiers and spirals, ascending to the rafters in three-dimensional circular pavilions and transforming the 55,000-square-foot space of Manhattan’s Wade Thompson Drill Hall at the Armory. “It’s more of a happening than an exhibit,” says Ms. Warren, comparing the project to an indoor version of Christo and Jeanne-Claude’s “The Gates,” a site-specific project featuring 23 miles of gates positioned around Central Park in 2005.

A 2010 Quilting in America survey, conducted by TNS Global Inc., indicated that quilting is now a $3.6 billion industry, with upwards of 21 million quilters active in the U.S. alone.

The auction market for quilts has fallen somewhat from its peak about 20 years ago, when quilts became extremely popular with Americana collectors. In 1987, a Baltimore Album Quilt dating to 1840 sold for $176,000 at Sotheby’s; a pictorial Civil War-era quilt sold for $264,000 at the auction house a few years later.

Though the market has cooled off somewhat since then, a rare, beautiful quilt in perfect condition could still fetch $100,000 or more today, says Nancy Druckman, Sotheby’s folk art specialist.

By presenting the exhibit free to the public, the Folk Art Museum hopes to attract new audiences. “This should not be looked at solely as a quilt exhibition,” says Ms. Warren. “The design is so extraordinary that it should appeal to those with an interest in design and graphics.”

A VORTEX QUILT by an unidentified American, 1890-1910, from Joanna Rose’s collection, part of the Armory show.